Noise robustness of persistent homology

across filtrations and signatures

Renata Turkeš

Young Topologists Meeting, July 8, 2021

Motivation

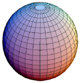
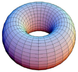
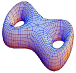
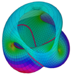
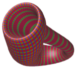

Persistent homology is a noise-robust topological summary.
Well...

Persistent homology

Homology & Betti numbers β

Homology captures information about k-dimensional cycles:

- connected components (0-dimensional homology)
- holes (1-dimensional homology)
- voids (2-dimensional homology)
- ...

β	Sphere	Torus	Two-holed torus	Projective plane	Klein bottle
					
β_0	1	1	1	1	1
β_1	0	2	4	1	2
β_2	1	1	1	1	1

Persistent homology = Homology of data

Persistent homology (PH), describes the shape of an object (a point cloud, an image, a time series, a network, etc), i.e., it captures information about k-dimensional cycles which persist across different scales $r \in \mathbb{R}$.

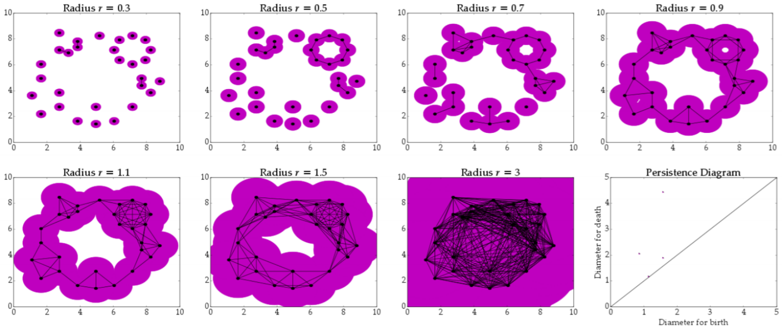


Figure taken from Munch, Elizabeth, *A user's guide to topological data analysis*, Journal of Learning Analytics 4.2 (2017): 47-61.

The black and the noisy red circle have similar persistence diagrams.

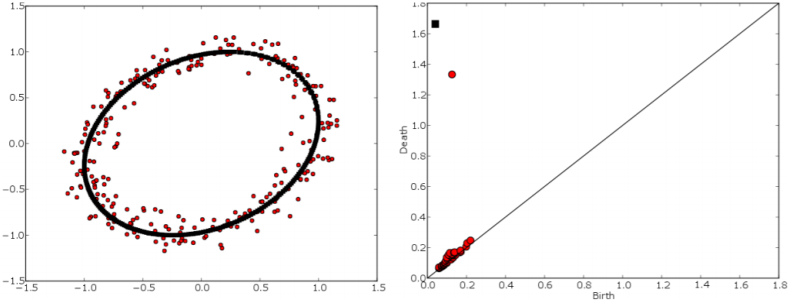


Figure taken from Munch, Elizabeth, *A user's guide to topological data analysis*, Journal of Learning Analytics 4.2 (2017): 47-61.

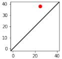
Noise robustness of persistent homology across filtrations and signatures

Motivation


Persistent homology is a noise-robust topological summary.
Well...

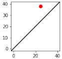
Persistent homology pipeline

image	$\longrightarrow$	persistence diagram



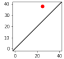
Persistent homology pipeline

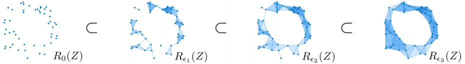
image	$\rightarrow$	filtration	$\rightarrow$	persistence diagram



Persistent homology pipeline

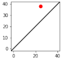
image	$\rightarrow$	filtration	$\rightarrow$	persistence diagram





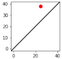
Persistent homology pipeline

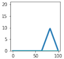
image	→	filtration	→	persistence diagram



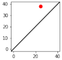
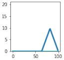
Persistent homology pipeline

image	→	filtration	→	persistence diagram	→	persistence signature



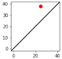


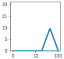
Persistent homology pipeline

image	→	filtration	→	persistence diagram	→	persistence signature
						

Persistent homology pipeline

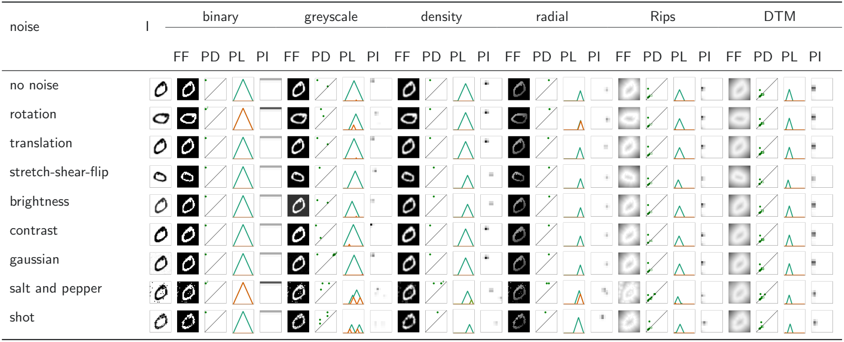
image	→	filtration	→	persistence diagram	→	persistence signature



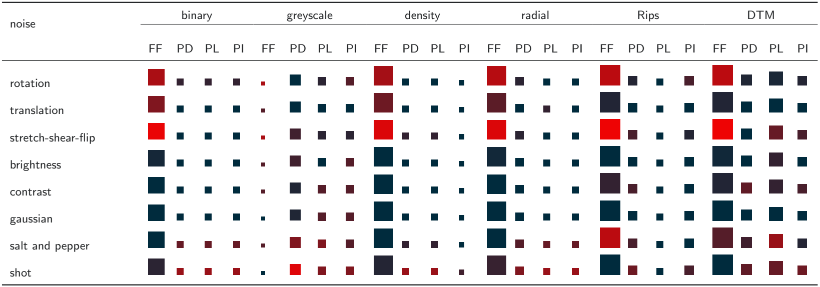


!!! Stability theorem: $d(PS(\phi), PS(\psi)) \leq c\|\phi - \psi\|_p$

Noise robustness of PH



Noise robustness of PH features in a classification task



node size = accuracy on the non-noisy test data

node color = drop in SVM classification accuracy when the test dataset is noisy, compared to the non-noisy test set (red indicates significant performance loss)

Noise robustness of PH features in a classification task

For SVM trained on non-noisy and tested on noisy images, there is at least a 30% drop in accuracy compared to non-noisy test data, for at least 0- or 1-dimensional PH, for at least one of the considered signatures:

- rotation and translation: radial
- stretch-shear-flip: radial, Rips, DTM
- brightness and contrast: greyscale
- gaussian: greyscale
- salt and pepper: binary, greyscale, density, radial, Rips, DTM
- shot: binary, greyscale, density, radial, Rips, DTM,

often varying across PDs, PLs and PIs.

Take-aways

- noise sensitivity of PH is influenced by the choice of filtration and persistence signature (input and output of PH)
- PH features are not always robust under any type of noise in a classification task

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