

# Noise robustness of persistent homology across filtrations and signatures

Renata Turkeš Young Topologists Meeting, July 8, 2021

## Motivation

Persistent homology is a noise-robust topological summary.

Well...

Persistent homology

Homology captures information about k-dimensional cycles:

- connected components (0-dimensional homology)
- holes (1-dimensional homology)
- voids (2-dimensional homology)

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$\beta$	Sphere	Torus	Two-holed torus	Projective plane	Klein bottle
		0			
$\beta_0$	1	1	1	1	1
$\beta_1$	0	2	4	1	2
$\beta_2$	1	1	1	1	1

#### Persistent homology = Homology of data

Persistent homology (PH), describes the shape of an object (a point cloud, an image, a time series, a network, etc), i.e., it captures information about k-dimensional cycles which persist across different scales  $r \in \mathbb{R}$ .



Figure taken from Munch, Elizabeth, A user's guide to topological data analysis, Journal of Learning Analytics 4.2 (2017): 47-61.

The black and the noisy red circle have similar persistence diagrams.



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Persistent homology is a noise-robust topological summary.

Well...

image	 persistence
	diagram





image	$\rightarrow$	filtration				$\rightarrow$	persistence diagram	
6		1-6	ŀ	6	6			





image $\rightarrow$ filtration	$\rightarrow$	persistence $\rightarrow$ diagram	persistence signature
61666			20 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5

image	$\rightarrow$	filtration	$\rightarrow$	$\begin{array}{ll} {\sf persistence} & \rightarrow \\ {\sf diagram} \end{array}$	persistence signature
6		16666			20 10 5 0 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
					persistence diagrams
					persistence landscapes

image	$\rightarrow$	filtration	$\rightarrow$	persistence $ ightarrow$ diagram	persistence signature
6		16666			
					persistence diagrams
æ					persistence landscapes

!!! Stability theorem:  $d(PS(\phi), PS(\psi)) \leq c \|\phi - \psi\|_{P}$ 

## Noise robustness of PH

noise	binary	greyscale	density	radial	Rips	DTM
	FF PD PL PI					
no noise	00//	0 / ^	0 / ^ •	0 / 🔥		
rotation	▫▫∕∧⁻	0 / 🔥 🖞	o / A ·			
translation	00//	0 / ^ *	0 / ^ •	0 Ž 🔥		
stretch-shear-flip	00//	o .∕ ∧ .	o ∕ ∧ ▪	۵Ż۸		
brightness	00//	0 / ^	0 / ^ -	0 Z _		
contrast	00//	0 / /	0 / ^ •	ΟŽ		
gaussian	00//	0 / _ `	0 / ^ •	0 Z A		
salt and pepper	00/	0 7 🔥 👘	0 / 🛆 -	0 / 🔥		<ul> <li>✓ ▲</li> </ul>
shot	00//	0 🥬 👬	0 / 🔨	0 Z 🔥		<ul> <li>✓ ∧</li> </ul>

#### Noise robustness of PH features in a classification task

noise	binary				greyscale				density			radial			Rips				DTM					
	FF	PD	PL	PI	FF	PD	PL	ΡI	FF	PD	PL	ΡI	FF	PD	PL	ΡI	FF	PD	PL	ΡI	FF	PD	PL	ΡI
rotation																								
translation											•	•				•			•					
stretch-shear-flip		•	•	•	•							•			•	•			•					
brightness		•	•	•	•						•	•			•	•			•					
contrast		•	•	•	•						•	•			•	•			•					
gaussian	2	•	•	•	•						•	•			•	•			•					
salt and pepper		•	•	•	•					•	•	•			•	•			•				_	
shot					•							•							•					

node size = accuracy on the non-noisy test data

node color = drop in SVM classification accuracy when the test dataset is noisy, compared to the non-noisy test set (red indicates significant performance loss)

## Noise robustness of PH features in a classification task

For SVM trained on non-noisy and tested on noisy images, there is at least a 30% drop in accuracy compared to non-noisy test data, for at least 0- or 1-dimensional PH, for at least one of the considered signatures:

- rotation and translation: radial
- stretch-shear-flip: radial, Rips, DTM
- brightness and contrast: greyscale
- gaussian: greyscale
- salt and pepper: binary, greyscale, density, radial, Rips, DTM
- shot: binary, greyscale, density, radial, Rips, DTM,

often varying across PDs, PLs and PIs.



- noise sensitivity of PH is influenced by the choice of filtration and persistence signature (input and output of PH)
- PH features are not always robust under any type of noise in a classification task

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